

Optimal Order Quantity Estimation in a Manufacturing Supply Chain

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Abstract

Production managers constantly seek ways to minimise waste and costs, so as to maximise profit. However, constraints such as budget, machine, manpower and storage capacities tend to make these optimisation tasks complex. Therefore, this study proposes a profit maximisation and optimal order quantity estimation model, considering various constraints. The study compares two optimisation methods, namely machine learning (Bayesian Optimisation) and nonlinear programming (PuLP Optimisation). The optimisation methods were applied to a small bottled water supply chain that produces bottled water of various sizes. The optimal order quantities from the Bayesian Optimisation for the 33cl, 50cl, 75cl and 150cl bottled water products are 300 packs, 483 packs, 150 packs and 33 packs, respectively, giving a maximum profit of ₦1,487,884. On the other hand, the optimal order quantities from the PuLP Optimisation for the 33cl, 50cl, 75cl and 150cl bottled water products are 295 packs, 519 packs, 177 packs and 91 packs, respectively, giving a maximum profit of ₦1,967,499. Though the PuLP Optimisation provided a higher profit, some of its order quantity estimates were above the average demand for the products. This could lead to product overstocking, and subsequent waste in the form of overproduction or excess inventory. On the other hand, though the Bayesian Optimisation was more computationally expensive, its order quantity estimates were less than or equal to the average demand for the products. Therefore, in the context of this study, the Bayesian Optimisation can be deemed to be better than its PuLP Optimisation counterpart. The study is significant to production management in general, and bottled water supply chains in particular, because it proposes an order quantity estimation and profit maximisation model, as well as a comparison of various methods for solving the model.

Keywords: Bottled Water, Bayesian Optimisation, PuLP Optimisation, Order quantity, EOQ, Supply Chain.

1. Introduction

Manufacturing supply chains are the networks that link suppliers of raw materials, manufacturers, distributors and consumers of products. They are the means by which raw materials are refined and distributed to customers (Kunovjanek et al., 2022; Naghshineh & Carvalho, 2022). Orders are extremely common in manufacturing supply chains. Through orders, a segment of a manufacturing supply chain communicates its requirements to its preceding neighbouring segment. Common types of orders within manufacturing supply chains are purchase orders, production orders, work orders, sales orders, transfer orders, maintenance orders, repair orders, return orders, requisition orders and stock or inventory orders. Purchase orders are created during procurement of raw materials or components from suppliers. Production orders are issued to initiate the manufacturing process. Work orders are similar to production orders but are mainly used for particular, detailed production process tasks. Sales orders are generated when a customer places an order for finished goods. Transfer orders are used to move materials or products between different facilities or locations within a company, such as from a warehouse to a production facility. Maintenance orders are issued to carry out maintenance activities on equipment or machinery within the production process, thereby aiding smooth functioning of production assets and downtime prevention. Repair orders are generated when products are returned by customers for repairs or when internal assets require fixing. Return orders are created for goods that need to be returned, either from customers i.e. return of sold products or from production back to inventory e.g. defective parts.

Requisition orders are used internally to request materials or parts needed for production. Stock or inventory orders are issued to manage inventory replenishment, especially for materials held in warehouses. Whether they are purchase orders, production orders, raw materials or finished products, each segment of a manufacturing supply chain usually requires transfer of items, to or from the preceding segment.

Optimisation is paramount in any manufacturing organisation that needs to reduce waste and costs, in order to maximise profit. The optimisation operation is usually led by managers in various echelons of manufacturing supply chains, who gather feedback data from supply chain personnel, as well as product consumers (Wofuru-Nyenke et al., 2023). These data are usually transformed into useful information and insights that can aid the managers in determining the best ways to manufacture and distribute products. As a matter of fact, managers are usually concerned about selecting the best suppliers of raw materials and services (Wofuru-Nyenke, 2023a); accurate forecasting of product demand (Wofuru-Nyenke & Briggs, 2022; Wofuru-Nyenke, 2022); utilising the most efficient methods and equipment (Wofuru-Nyenke, 2024a, 2024d; Wofuru-Nyenke, 2021a, 2023b; Wofuru-Nyenke & Okere, 2025); optimal scheduling of production activities (Wofuru-Nyenke, 2024e); selecting the most feasible, reliable and profitable product designs (Ugoji et al., 2022; Wofuru-Nyenke, 2024b; Wofuru-Nyenke, 2020, 2024f); and optimally locating facilities and distributing products (Wofuru-Nyenke, 2024c). Therefore, several authors and researchers have made attempts at developing models that aid the manufacturing supply chain optimisation process (Ahmad et al., 2022; Awudu et al., 2024; Ebrahimi & Bagheri, 2022; Islam et al., 2022; Ravindran et al., 2023; Sun et al., 2022; Verma et al., 2024; Zerafati et al., 2022). Despite several attempts at developing optimisation models, there is a lack of generic models and associated model solutions for supply chain profit maximisation considering budget, storage capacity and service level constraints.

Therefore, the aim of this study is to propose a profit maximisation and waste reduction model for determining the optimal order quantities of different products in a supply chain echelon, while considering various constraints, such as budget, storage capacity, and service level. The study compares two approaches of solving the model, namely machine learning (Bayesian Optimisation) and nonlinear programming (PuLP Optimisation). The following section presents the equations and constraints involved in the mathematical formulation of the model, as well as the flowcharts that display the logic of the Python implementation of the optimisation techniques.

2. Methodology

This study proposes a mathematical model for maximising profit, through the optimisation of order quantities of products in the factory warehouse echelon of a supply chain, considering demand, costs, safety stock, lead time, budget, and storage constraints. Figure 1 shows the interrelationship between the various segments and echelons of the small bottled water manufacturing supply chain, whose optimal order quantities are being estimated. From Figure 1, the small supply chain network starts with suppliers that deliver raw materials to the water bottling factory, after receiving the purchase order. In turn, the factory sends finished products to the factory warehouse, after receiving the production order. The production order usually contains the order quantities, Q_p , of the various products handled by the supply chain. Finally, the factory warehouse satisfies product demand emanating from consumers or distribution warehouses. This study is concerned with determining the optimal values of Q_p that maximise profit and reduce waste. The optimisation problem is formulated as a profit maximisation problem and the objective function can be expressed as

$$\max_{Q_p} \text{Profit}(Q_p) \quad \text{subject to budget, capacity and service level constraints} \quad (1)$$

where,

Q_p is the order quantity of product p ,

Profit is the difference between the revenue from sales of the product and the total cost.

In fact, the total cost comprises of ordering, holding, and purchasing costs. Therefore, the profit function can be expressed as

$$\text{Profit} = \sum_{p \in P} (R_p - H_p - O_p - P_p) \quad (2)$$

where,

P is the set of products,

R_p is the revenue from product, p ,

H_p is the holding cost,

O_p is the ordering cost

P_p is the production cost of product, p .

The revenue, R_p , from sales of product, p , can be expressed as

$$R_p = S_p \cdot \min(Q_p, D_p) \quad (3)$$

where,

S_p is the selling price per unit,

Q_p is the order quantity,

D_p is the average monthly demand of product, p , at the factory warehouse.

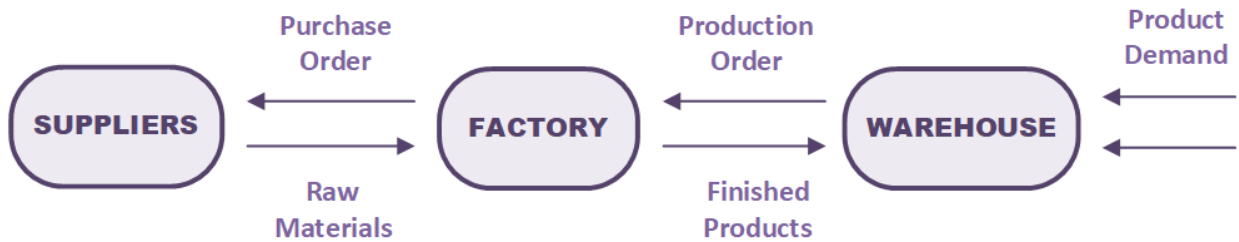


Figure 1: Interrelationship between segments of the bottled water supply chain.

Assuming the average inventory level is $\frac{Q_p}{2}$, the holding cost, H_p , of product p , can be expressed as

$$H_p = \frac{Q_p}{2} \cdot H_p \quad (4)$$

where,

Q_p is the order quantity of product p .

Assuming the total number of orders placed is $\frac{D_p}{Q_p}$, the ordering cost, O_p , of product p , including variable and fixed order cost and can be expressed as

$$O_p = \frac{D_p}{Q_p} \cdot O_p \quad (5)$$

where,

D_p is the average monthly demand of product, p , at the factory warehouse,

Q_p is the order quantity of product p .

While the production cost of product p , P_p , can be expressed as

$$P_p = Q_p \cdot C_p \quad (6)$$

where,

Q_p is the order quantity of product p ,

C_p is the production cost per unit of product p .

If by make-or-buy decisions, the factory warehouse decides to use their supply chain manufacturing plant to make the stored finished products, then P_p is referred to as the production cost of product p . However, if the factory warehouse decides to buy the stored finished products from a different manufacturer or factory, then P_p is the purchasing cost.

From the foregoing, by substituting equations 3, 4, 5 and 6 in equation 2, the total profit across all products can be expressed as

$$\text{Profit} = \sum_{p \in P} \left(S_p \cdot \min(Q_p, D_p) - \frac{Q_p}{2} \cdot H_p - \frac{D_p}{Q_p} \cdot O_p - Q_p \cdot C_p \right) \quad (7)$$

Subject to the following constraints

1. Production/Purchasing Budget Constraint

This constraint demands that the total production cost of all products should not exceed the available production budget, B , and it can be expressed as:

$$\sum_{p \in P} P_p \leq B \quad (8)$$

2. Storage and Ordering Budget Constraint

This constraint demands that the total storage and ordering costs of all products should not exceed the available storage and ordering budget, B_{so} , and it can be expressed as:

$$\sum_{p \in P} (H_p + O_p) \leq B_{so} \quad (9)$$

3. Storage Capacity Constraint

This constraint demands that the total inventory units across all products should not exceed the storage capacity, SC , and it can be expressed as:

$$\sum_{p \in P} Q_p \leq SC \quad (10)$$

4. Reorder Point Constraint

The reorder point (ROP) is the inventory level at which an order should be placed to avoid stockouts. The ROP constraint demands that the order quantity, Q_p for each product should be greater than or equal to its ROP, and it can be expressed as:

$$Q_p \geq \text{ROP}_p \quad \forall p \in P \quad (11)$$

The reorder point for a particular product (ROP_p) can be expressed as (Chopra & Meindl, 2016):

$$\text{ROP}_p = \mu_p L_p + z \cdot \sigma_p \sqrt{L_p} \quad (12)$$

where,

$\mu_p = \frac{D_p}{30}$ is the average daily demand for product, p , at the factory warehouse,

L_p is the lead time (days) for product p ,

z is the safety factor corresponding to the desired service level,

σ_p is the standard deviation of daily demand for product, p , at the factory warehouse.

This study applied the Bayesian Optimisation algorithm implemented in Python programming language, through the Bayesian-optimisation library. The algorithm is a sequential optimisation algorithm that globally optimises an objective function, through the utilisation of an objective function probability model to select hyperparameters for evaluation in the true objective function, in order to obtain the optimum value of the function (Garnett, 2023). The nonlinear profit maximisation model was also solved by linearising the nonlinear components, and using the PuLP Optimisation library for obtaining optimal values of product order quantities. The PuLP Optimisation uses the Economic Order Quantity (EOQ) formula for determining the optimal order quantity. The EOQ for each product, p , can be expressed as (Chopra & Meindl, 2016):

$$\text{EOQ}_p = \sqrt{\frac{2D_p O_p}{H_p}} \quad (13)$$

where,

D_p is the average monthly demand for product, p , at the factory warehouse,

O_p is the ordering cost of product, p ,

H_p is the holding cost of product, p .

The optimum from both methods were compared, and the results of the comparison were presented and discussed in the results and discussion section. For comparing the maximised profits obtained from the two optimisation methods, the percentage difference formula was used. This formula can be expressed as

$$\% \text{ Difference (Profit)} = \frac{|P_{BO} - P_{PO}|}{\left(\frac{P_{BO} + P_{PO}}{2}\right)} \times 100 \quad (14)$$

where,

P_{BO} is the profit from Bayesian Optimisation,

P_{PO} is the profit from PuLP Optimisation.

The optimisation algorithms were applied to a small water bottling company which produces four products namely, 33cl, 50cl, 75cl and 150cl bottled water. The data from the water bottling company are shown in Table 1. From Table 1, the average monthly demand for the 33cl, 50cl, 75cl and 150cl bottled water products are 300, 600, 150 and 50 packs, respectively. Also, the costs of ordering raw materials for the 33cl, 50cl, 75cl and 150cl bottled water products are ₦500, ₦400, ₦600 and ₦600 respectively. Moreover, the cost of storing the bottled water product is ₦ 100 for each category of bottled water product. Again, the selling prices of the 33cl, 50cl, 75cl and 150cl bottled water products are ₦1,500, ₦1,800, ₦2,000 and ₦3,000 per pack, respectively.

Table 1: Data from water bottling company.

Data	Products			
	33cl	50cl	75cl	150cl
Average demand (Packs/Month)	300	600	150	50
Ordering cost (₦)	500	400	600	600
Holding cost (₦)	100	100	100	100
Selling price (₦)	1500	1800	2000	3000
Production cost per unit (₦)	20	30	40	40
Daily standard deviation (Packs)	5	5	5	5
Lead time (Days)	21	21	21	21
Budget (₦)	100,000			
Storage capacity (Packs)	1,000			
Service level	0.9			

The production cost per unit of the 33cl, 50cl, 75cl and 150cl bottled water products are ₦20, ₦30, ₦40, ₦40, respectively. The daily standard deviation and lead time for the 33cl, 50cl, 75cl and 150cl bottled water products are 5 packs and 21 days, respectively. Finally, the budget, storage capacity and service level of the water bottling company are ₦100,000, 1,000 packs and 0.9, respectively. The service level represents the ability of the warehouse to completely meet customer demand on time. In this study, the service level ranges from 0, which is low, to 1, which is high. The justification for selecting a service level of 0.9 is that the optimal order quantities predicted at that service level will still be the best if the service level drops. Figure 2 shows the flowchart of the optimisation logic utilised by the Bayesian Optimisation algorithm, while Figure 3 shows the flowchart of the PuLP Optimisation logic.

3. Results and Discussion

The Bayesian Optimisation algorithm was used to obtain the optimal order quantities of each of the products produced by the water bottling company, as well as the optimal solution of the profit maximisation problem. The optimisation was run with the “n_iter” and “init_points” parameters set to 200 steps and 300 steps, respectively, giving a total of 500 iterations. The results indicated that in order to efficiently meet customer demand, the optimal order quantity for

the 33cl, 50cl, 75cl and 150cl bottled water products are 300 packs, 483 packs, 150 packs and 33 packs, respectively. Figure 4 shows the plot of Profit versus Iteration, and the maximum profit that can be obtained from using these order quantities.

In Figure 4, the maximum profit occurs at iteration 348 and the value is ₦1,487,884. By linearising the nonlinear components of the objective function, the PuLP Python library was also used to solve the nonlinear profit maximisation problem, and obtain optimal order quantities for each of the products of the water bottling company. The results indicated that in order to efficiently meet customer demand, the optimal order quantity for the 33cl, 50cl, 75cl and 150cl bottled water products are 295 packs, 519 packs, 177 packs and 91 packs, respectively. Moreover, the maximised profit when these order quantities are utilised is ₦1,967,499. Figure 5 shows a comparison between the demand and the optimal order quantities from each of the optimisation methods.

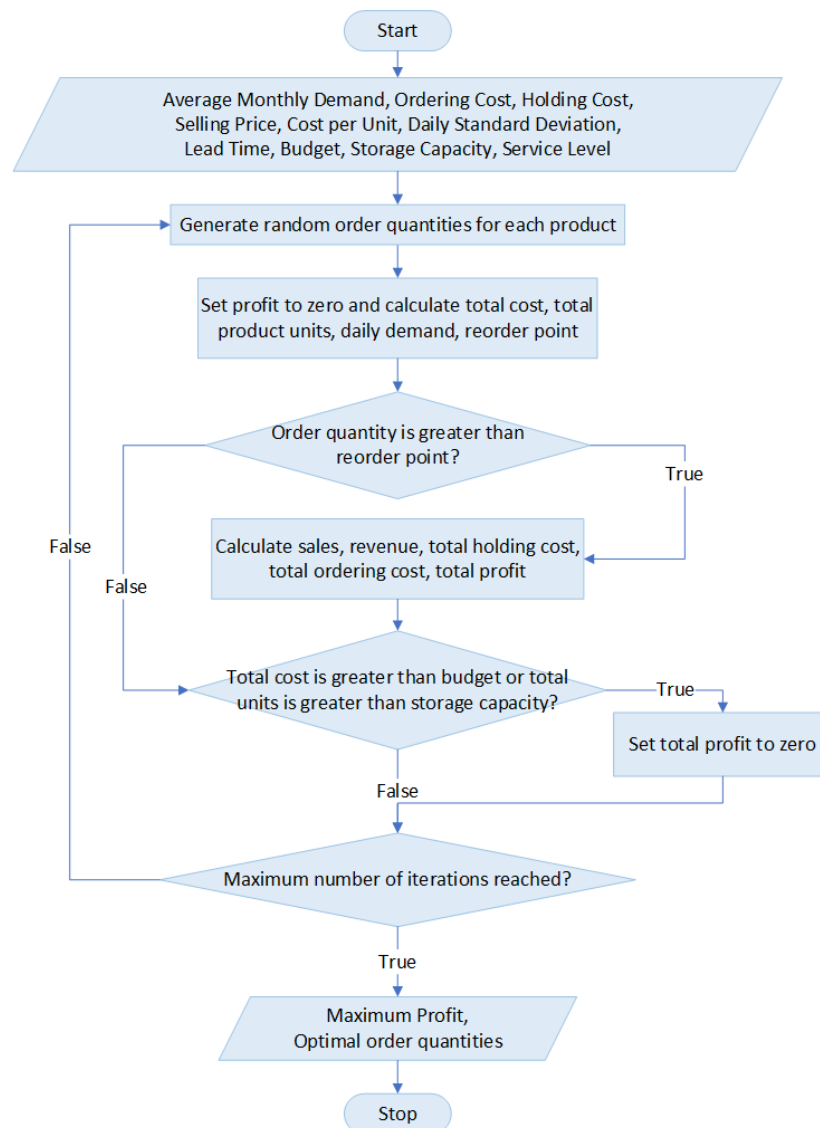


Figure 2: Flowchart of the Bayesian Optimisation logic for optimising order quantities.

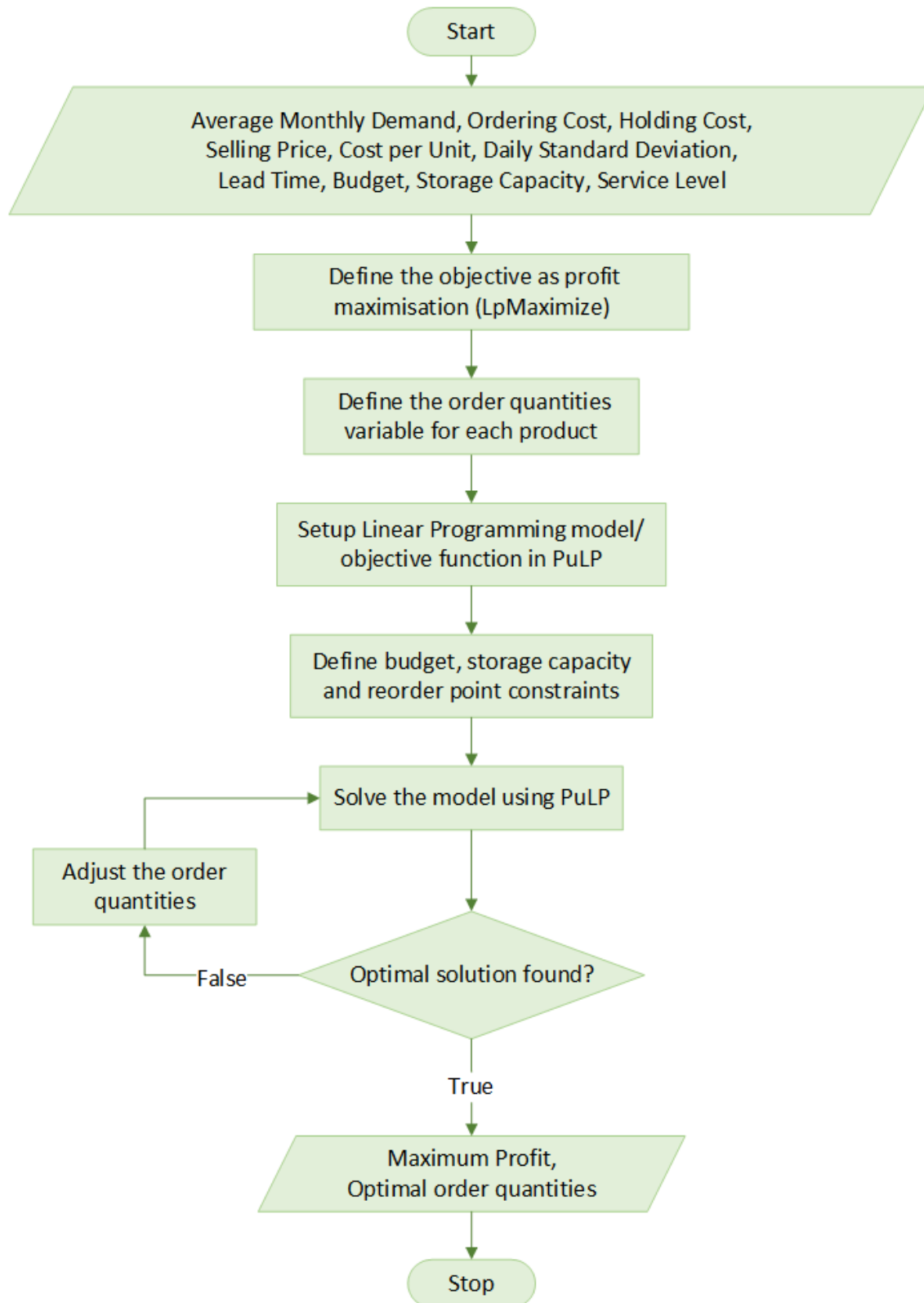


Figure 3: Flowchart of the PuLP Optimisation logic for optimising order quantities.

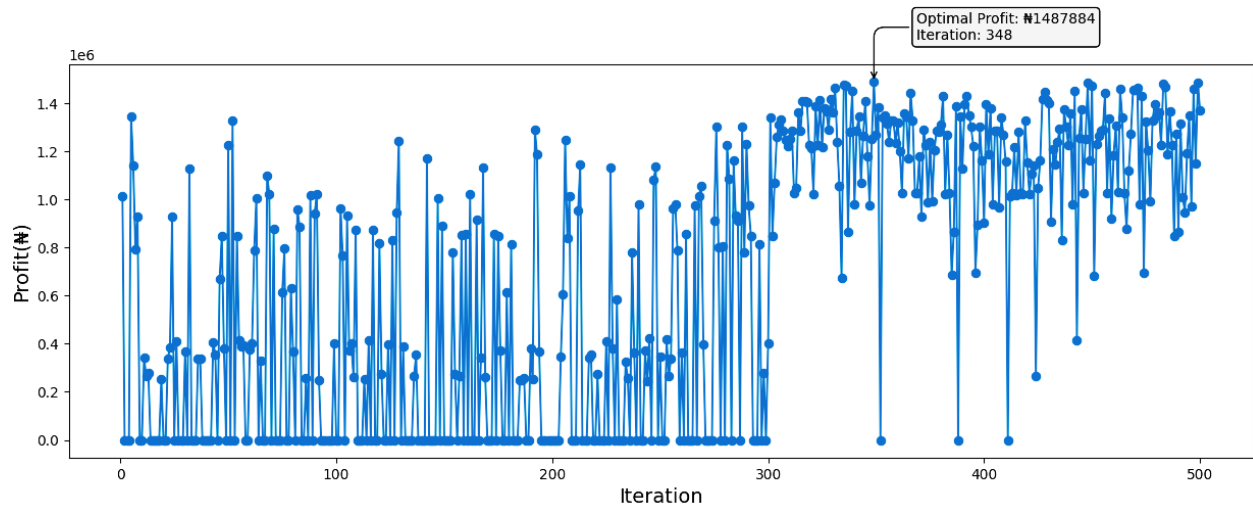


Figure 4: Plot of Profit versus Iteration for the bottled water products.

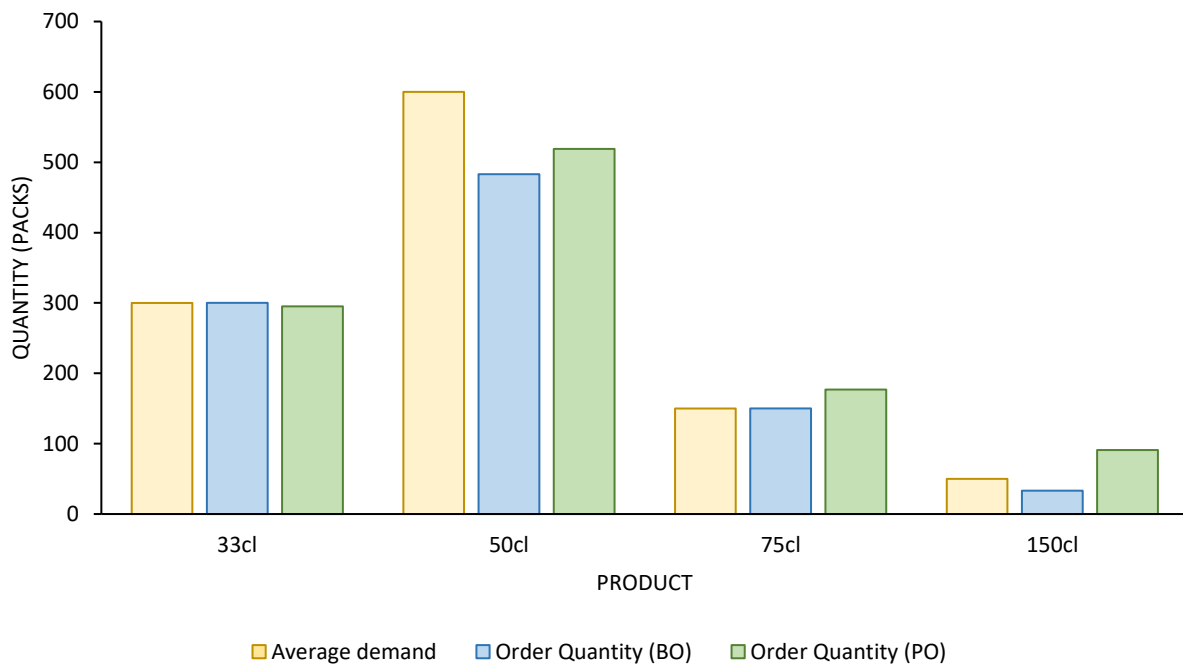


Figure 5: Comparison of average demand and optimal order quantities from Bayesian Optimisation (BO) and PuLP Optimisation (PO).

The percentage difference between the maximised profits from the Bayesian Optimisation and PuLP Optimisation is 27.76%. This means that the two profits differ by 27.76%, relative to their average. Figure 6 shows a comparison of the total profit obtained from the Bayesian and PuLP Optimisation methods.

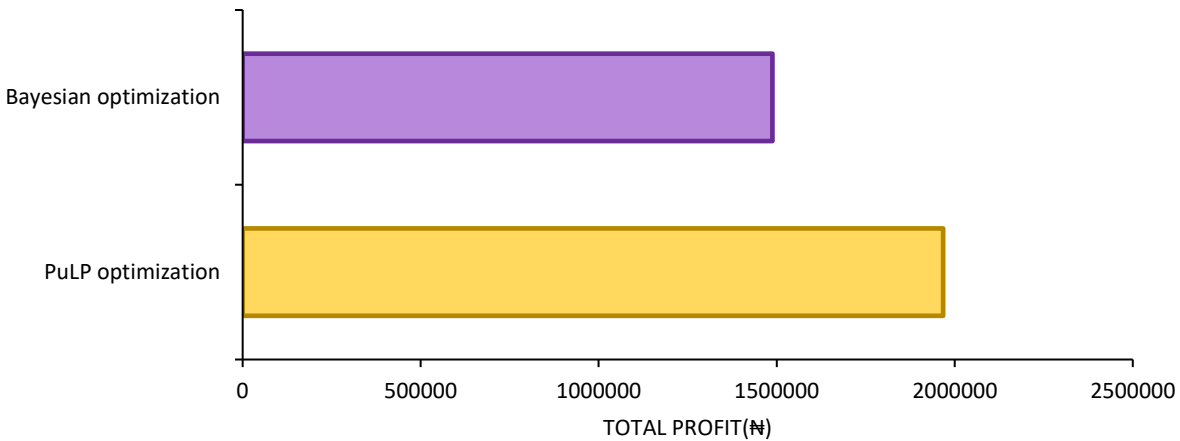


Figure 6: Comparison of total profit from Bayesian Optimisation and PuLP Optimisation.

In Figure 6, the profit from the PuLP Optimisation is greater than that from the Bayesian Optimisation. By analysing Figures 3 and 4, it can be observed that the total optimal order quantities obtained from the Bayesian Optimisation method are less than those obtained from the PuLP Optimisation method, thereby justifying the larger total profit from the PuLP Optimisation. Probably, by increasing the number of iterations in the Bayesian Optimisation method, better optimal order quantities and larger total profit could be obtained. This can be achieved by setting the “n_iter” and “init_points” parameters within the Bayesian Optimisation code to higher values. However, the major drawback of increasing the number of iterations in Bayesian Optimisation, is the apparent computational expensiveness of the optimisation algorithm. This means that as the number of iterations of the Bayesian Optimisation increases, the model consumes more resources in order to determine the optimal values. Moreover, the results from the Bayesian Optimisation are dynamic. This means that the results change, based on different input data to the objective function or optimisation model.

4. Conclusion

This study proposes a mathematical model for maximising profit, through the optimisation of order quantities of products in the factory warehouse segment of a manufacturing supply chain. The model takes demand, costs, safety stock, lead time, budget, and storage constraints into consideration, while determining the optimal values. The model was solved by using two optimisation methods namely, Bayesian and PuLP Optimisation, both implemented in Python. From the results, both methods perform well at determining the optimal order quantities of bottled water products. However, in comparing both methods, the Bayesian Optimisation method is more computationally expensive, requiring more resources to determine the optimum. Also, the optimal order quantities generated by the PuLP Optimisation tend to be higher than the average demand, which could lead to waste in the form of overproduction or excess inventory, within the supply chain facilities. Since the aim of the optimisation is to determine optimal order quantities that maximise profit while reducing waste, the Bayesian Optimisation can be deemed to be superior to the PuLP Optimisation, within the context of this study.

The research has shown how a machine learning technique (Bayesian Optimisation), as well as a nonlinear programming technique (PuLP Optimisation) can be used for optimal order quantity estimation and profit maximisation. The study is significant to production management in general, and bottled water supply chains in particular, because it proposes an order quantity estimation and profit maximisation model, as well as a comparison of various methods for solving the model. Further research can investigate the efficacy of other optimisation methods at maximising profit while reducing waste. Therefore, optimisation methods such as genetic algorithms, simulated

annealing and particle swarm optimisation can be investigated, for optimal order quantity estimation and profit maximisation. Moreover, the methods investigated in this research utilised a deterministic average demand value. Further research can utilise a probability distribution for demand, to better capture the randomness in empirical demand data.

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